Prob. 2.

Show that maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$

Sol^n.

We know that $\sum_{i=1}^{n} d(v_i) = 2e$, e is the no. of edges.

$\therefore d(v_1) + d(v_2) + \cdots + d(v_n) = 2e$.

Since we know that that maximum degree of each vertex in a simple graph G can be $(n-1)$

Therefore we can write

$$e \leq \frac{(n-1) + (n-1) + \cdots + (n-1)}{2}$$

i.e, $$e \leq \frac{n(n-1)}{2}.$$

Thus maximum no. of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

Directed Graphs or Diagraphs.

A directed graph 'D' is a pair of two things $(V(D), E(D))$ where $V(D)$ is non-empty set of elements known as vertices and $E(D)$ is a family of ordered pair of vertices called arcs.

In other words, if each edge of the graph 'G' has a direction then the graph is called Directed Graph or Digraph.

Figures 5(a) & 5(b) represent two diagraphs in which the vertex set $V = \{v_1, v_2, v_3, v_4\}$ and directed edge set $E = \{e_1, e_2 \cdots e_7\}$

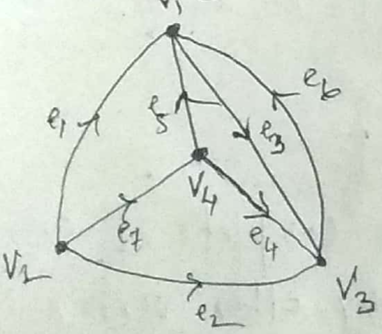


Fig - 5(a)

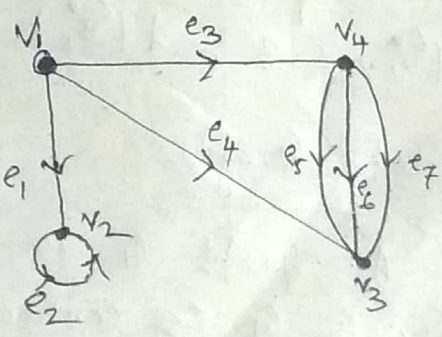


Fig - 5(b)

Scanned by CamScanner

The edges in Fig 5(a) are given by

$e_1 = (v_1, v_2)$, $e_2 = (v_2, v_3)$, $e_3 = (v_1, v_3)$, $e_4 = (v_4, v_3)$

$e_5 = (v_4, v_1)$, $e_6 = (v_3, v_1)$, $e_7 = (v_2, v_4)$

and the edges in Fig 5(b) are

$e_1 = (v_1, v_2)$, $e_2 = (v_2, v_3)$, $e_3 = (v_1, v_4)$, $e_4 = (v_1, v_3)$

$e_5 = (v_4, v_3)$ $e_6 = (v_4, v_3)$, $e_7 = (v_4, v_3)$

In fig 5(a) e_3 and e_6 joining v_1 and v_3 having opposite direction and without loops, hence this is called simple arcs.

On the other hand 5(b) v_3 and v_4 are joined by three are e_5, e_6, e_7 with same direction; such arc are called multiple arcs.

Indegree and out degree

The indegree of a vertex v of a diagraph D is defined as the number of arcs which are incident into v and denoted by $d(\vec{v})$.

On the other hand, the outdegree of a vertex v is defined as the number of arcs which are incident out of v and denoted by $d(\overleftarrow{v})$

∴ Indegree of different vertices in fig 5(b) are

$d(\vec{v_1}) = 0$, $d(\vec{v_2}) = 2$, $d(\vec{v_3}) = 4$, $d(\vec{v_4}) = 1$

and Outdegree are

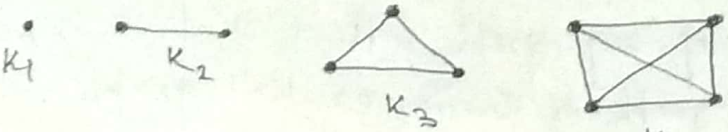
$d(\overleftarrow{v_1}) = 3$, $d(\overleftarrow{v_2}) = 1$, $d(\overleftarrow{v_3}) = 0$, $d(\overleftarrow{v_4}) = 3$.

Complete Graph.

A graph G is said to be complete if every vertex in G is connected to every other vertex in G.

Scanned by CamScanner

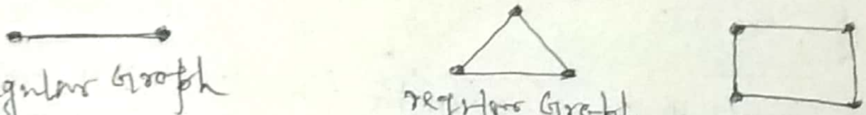
The complete graph with n vertices is denoted by K_n.



K_1 K_2 K_3 K_4

Regular Graph.

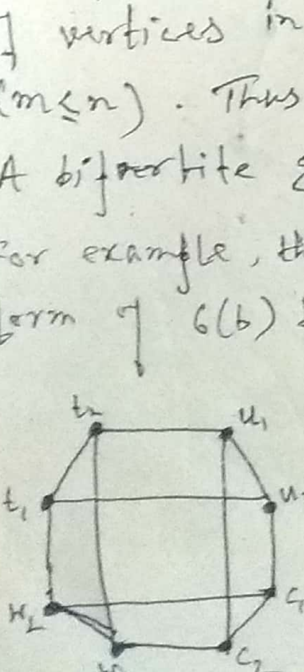
If the degree of each vertex is same, r (say) in any graph G then it is called a regular graph of degree r.



Regular Graph of degree 1 — regular Graph of degree 2 — Regular Graph of degree 2

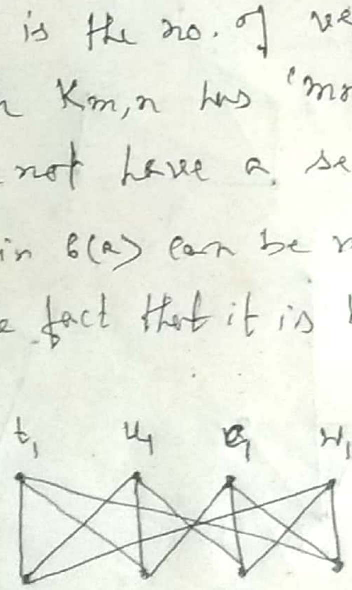
Bipartite Graph.

A graph G is said to be Bipartite if its vertex 'v' can be partitioned into two disjoint subsets X_1 and X_2 such that each edge of G connected a vertex v_{x_1} of X_1 to v_{x_2} of X_2.
It is not necessary that every vertex in X_1 is adjacent to every vertex in X_2 - But if it happens then G is called complete bipartite graph.
The graph is denoted by $K_{m,n}$ where m is the no. of vertices in X_1 and n is the no. of vertices in X_2. $(m \leq n)$. Thus the graph $K_{m,n}$ has 'mn' edges.
A bipartite graph cannot have a self loop.
For example, the graph in 6(a) can be redrawn in the form of 6(b) to display the fact that it is bipartite.



6(a)

$K_{4,4}$

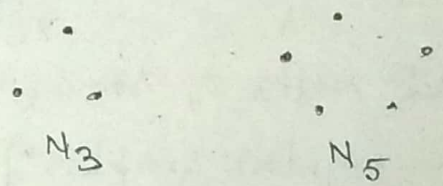


6(b)

$X_1 = \{t_1, u_1, c_1, w_1\}$ $X_2 = \{t_2, u_2, c_2, w_2\}$

Null Graph.

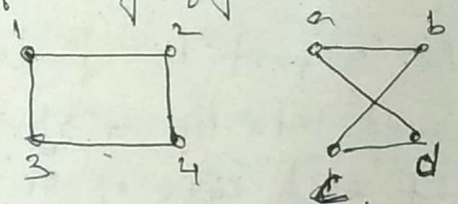
If the edge set of any graph with n-vertices is an empty set, then the graph is known as Null graph, denoted by N_n.



Isomorphism of Graphs.

Two graphs G and H are isomorphic (written $G \cong H$ or sometimes $G = H$) if there exists a one-to-one correspondence between their point sets which preserves adjacency.

The following figures is isomorphic



The one-one correspondence between the vertices are

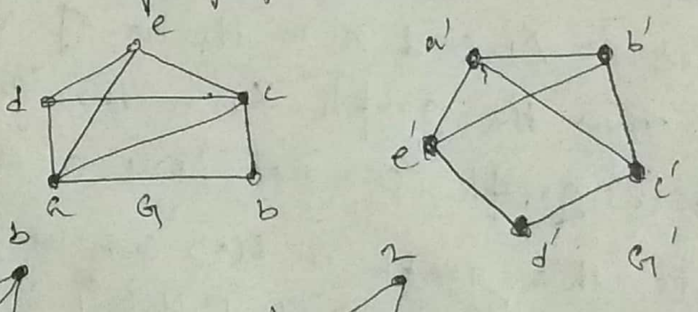
$1 \longrightarrow a \quad 2 \longrightarrow b, \; 3 \longrightarrow d, \; 4 \longrightarrow c$

Two isomorphic graphs must have same no. of vertices, edges, and equal no. of vertices with a given degree.

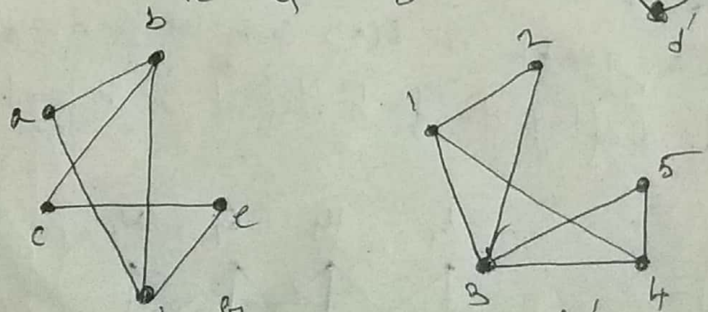
Prob. Are the following graphs G and G' are isomorphic? Justify:

Soln.

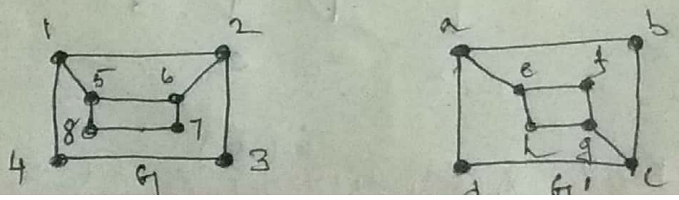
i)



ii)



iii)



Sol^n. (iii).

Both the graph G and G' contains 8 vertices ~~edges~~ and 10 edges. The no. of vertices of degree 2 in both the graphs are 4. Also the no. of vertices of degree 3 in both the graphs are 4.

For adjacency, consider the vertex 1 of degree 3 in G, it is adjacent to two vertices of degree 3 and one vertex of degree 2.

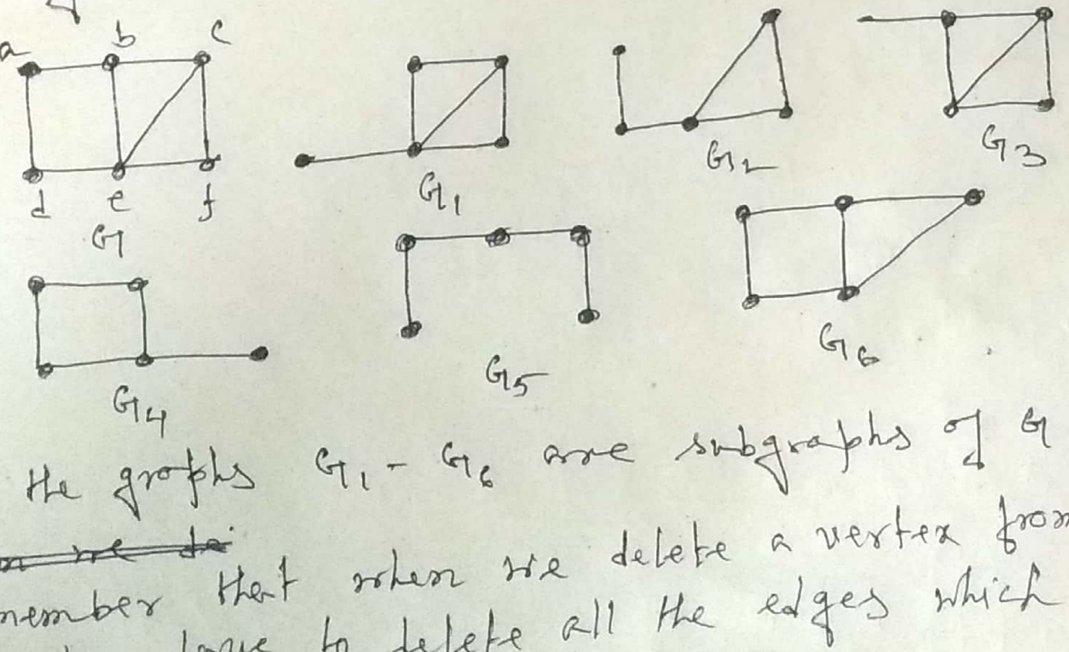
But in G' the vertices with degree 3 (a, c, e, g) has no adjacent two vertices of degree 3 and one vertex of degree 2.

Hence adjacency is not preserved.

Therefore the given curve ~~is~~ G & G' are not isomorphic.

Subgraph

A subgraph G' of a graph G is a graph having all its points and edges in G.



All the graphs $G_1 - G_6$ are subgraphs of G.

~~When we do~~ Remember that when we delete a vertex from G we also have to delete all the edges which contain the vertex.